Asymptotic Deconfinement in High-Density QCD

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We discuss QCD with two light flavors at large baryon chemical potential μ. Color superconductivity leads to partial breaking of the color SU(3) group. We show that the infrared physics is governed by the gluodynamics of the remaining SU(2) group with an exponentially soft confinement scale $\Lambda'_{\rm QCD} \sim \Delta \exp(-a\mu/(g\Delta))$, where $\Delta \ll \mu$ is the superconducting gap, g is the strong coupling, and $a = 2\sqrt{2}\pi/11$. We estimate that, at moderate baryon densities, $\Lambda'_{\rm QCD}$ is $\mathcal{O}(10~\text{MeV})$ or smaller. The confinement radius increases exponentially with density, leading to "asymptotic deconfinement." The velocity of the SU(2) gluons is small due to the large dielectric constant of the medium.

Introduction.—Soon after the discovery of asymptotic freedom in QCD [1] a hypothesis was put forward that, at high baryon densities, quarks (which are normally confined in hadrons by strong forces) are liberated, i.e., nuclear matter transforms into deconfined quark matter [2]. In recent years, our knowledge of dense quark matter has considerably expanded. We now understand that, in reality, dense matter shows more intricate features than in the original picture of [2]. In particular, quark matter at high densities exhibits the phenomenon of color superconductivity [3,4], which determines the symmetry of the ground state and the infrared dynamics.

The number of light quark flavors N_f turns out to play a crucial role. The simplest case is $N_f = 2$, where up and down quarks are massless and other quarks are neglected. The following picture emerges in perturbation theory, as well as in instanton-inspired models. The condensation of color antitriplet up-down diquarks breaks the color SU(3) down to an SU(2) subgroup. Thus, five of the original eight gluons acquire "masses" by the Meissner effect [4,5], similar to the Higgs mechanism. The remaining three gluons are massless (perturbatively). Because of Cooper pairing, the spectrum of quark excitations carrying SU(2) color charge has a gap Δ.

In order to understand the physics below the energy scale Δ we must examine the pure gluodynamics in the remaining unbroken SU(2) sector. As we shall see, the process of high-density "deconfinement" is quite nontrivial in this case: the quarks are *always* confined (assuming that SU(2) Yang-Mills theory confines), however, the confinement radius grows *exponentially* with increasing density. We shall also see that, at scales much shorter than the confinement radius, the dynamics of the SU(2) gluons is similar to electrodynamics in a dielectric medium with large refraction index.

The effective Lagrangian.—Below the scale Δ, we expect that the heavy (gapped) degrees of freedom decouple and the remaining fields can be described by a local effective Lagrangian. The absence of quarks carrying SU(2) charges below Δ implies that the medium is transparent to the SU(2) gluons: there is no Debye screening and Meissner effect for these gluons. Mathematically, the polarization tensor $\Pi^{\mu\nu}_{ab}(q)$ vanishes at $q = 0$, which can be checked by a direct calculation of Π at small q, as was done in Ref. [5]. The absence of Debye screening means that a static color charge inserted into the medium cannot be completely screened as it is in hot plasmas. This is easy to understand since all quarks carrying SU(2) color are bound into SU(2) singlet Cooper pairs. Analogously, the Meissner effect is absent because the superconducting currents, which are coherent motions of the condensate, cannot screen the magnetic field, since the condensate is SU(2) neutral. Thus, at first sight, it might seem that the quarks in the medium have no effect on the gluon effective Lagrangian, which must be simply $L = -F^2_{\mu\nu}/(4g^2)$, i.e., the SU(2) Yang-Mills Lagrangian with the coupling g matching the running coupling in the original theory at the scale Δ [6]. However, a closer look shows that the situation is somewhat more complicated and, in fact, more interesting.

Although a static SU(2) charge cannot be completely Debye screened by SU(2) neutral Cooper pairs, it can still be *partially* screened if the medium is polarizable, i.e., if it has a *dielectric constant* ϵ different from unity. If $\epsilon > 1$, then the Coulomb potential between two static color charges is $g^2/(\epsilon r)$; i.e., the gauge coupling is effectively reduced by a factor of $\epsilon^{1/2}$. As explained in more detail below, this is exactly the situation in the color-superconducting phase. Analogously the medium can, in principle, have a magnetic permeability $\lambda \neq 1$. (We denote the permeability by λ instead of the more common μ, since the latter symbol is already used for the chemical potential.) The dynamics of gluons is thus modified by the dielectric constant and the magnetic permeability of the medium. Hence, one needs to develop the theory of "gluodynamics of continuous media", which, as far as we know, has never been encountered before. This theory, in contrast to its U(1) counterpart (the electrodynamics

of continuous media), is an *interacting* theory.

Fortunately, even without explicit calculation we can already write the effective Lagrangian of SU(2) gluons from rather general arguments. It should satisfy the requirements of locality (since the quarks that have been integrated out have gaps) and gauge invariance. It does not need to be Lorentz invariant, since this invariance is already violated by the presence of the high-density medium, but it should be rotationally invariant and conserve parity. Thus the effective action at the scale Δ must have the following form

$$S_{\rm eff} = \frac{1}{g^2}\int d^4x \left(\frac{\epsilon}{2}\mathbf{E}^a\cdot\mathbf{E}^a - \frac{1}{2\lambda}\mathbf{B}^a\cdot\mathbf{B}^a\right), \qquad (1)$$

where $E_i^a \equiv F_{0i}^a$ and $B_i^a \equiv \frac{1}{2}\epsilon_{ijk}F_{jk}^a$. Higher-order corrections (in powers of fields and derivatives) to (1) are irrelevant for the infrared physics and have been neglected. The constants ϵ and λ in Eq. (1) have the meaning of the the dielectric constant and the magnetic permeability in the regime where the gluon fields are linear. The speed of gluons in this regime is $v = 1/\sqrt{\epsilon\lambda}$.

As we shall see, $\epsilon\lambda \gg 1$. This should be contrasted with the standard picture of the vacuum as a dielectric, in which case $\epsilon_0\lambda_0 = 1$ by Lorentz invariance. In other words, vacuum polarization effects (such as gluon, ghost, and $\mu = 0$ quark loops) in Eq. (1) have been absorbed into the running of g^2. The constants ϵ and λ are defined relative to those of the vacuum.

One can give a parametrically correct estimate of ϵ using a crude model, in which the Cooper pairs are represented by classical oscillators with the spring constant k. The characteristic scale for a Cooper pair "oscillator" is Δ; thus one can estimate $k \sim \Delta^3$. The polarizability of each oscillator is g^2/k. Multiplying by the density of the Cooper pairs, $\mu^2\Delta$, one finds $\epsilon - 1 \sim g^2\mu^2/\Delta^2$. This estimate is corroborated by the explicit calculation, presented later in this paper, where we obtain

$$\epsilon = 1 + \kappa = 1 + \frac{g^2\mu^2}{18\pi^2\Delta^2}, \qquad (2)$$

$$\lambda = 1. \qquad (3)$$

At high densities, the gap Δ is exponentially suppressed compared to the chemical potential μ [7],

$$\Delta = b\mu g^{-5}e^{-c/g}, \qquad c = \frac{3\pi^2}{\sqrt{2}}, \qquad (4)$$

where g is the gauge coupling at the scale μ, and b is some numerical constant. According to Eq. (2), $\kappa \gg 1$ and we can write

$$\epsilon \approx \frac{g^2\mu^2}{18\pi^2\Delta^2} \gg 1, \qquad (5)$$

which means that the dielectric constant of the medium is very large. Hence, the Coulomb potential between SU(2) color charges is greatly reduced. This can be interpreted as a consequence of the fact that the Cooper pairs have large size (of order $1/\Delta$) and so are easy to polarize. The magnetic permeability, in contrast, remains close to 1 due to the absence of mechanisms that would strongly screen the magnetic field.

The scale of confinement.—Once the effective Lagrangian (1) is obtained, one can use it to investigate the infrared dynamics of the gluons. One notices that (1) possesses a modified Lorentz symmetry in which the speed of light $c = 1$ is replaced by

$$v = \frac{1}{\sqrt{\epsilon}}. \qquad (6)$$

One can make this symmetry manifest by rescaling the time, the field A_0 and the coupling in Eq. (1),

$$x^{0\prime} = \frac{x^0}{\sqrt{\epsilon}}, \quad A_0^{a\prime} = \sqrt{\epsilon}A_0^a, \quad g' = \frac{g}{\epsilon^{1/4}}. \qquad (7)$$

After the rescaling (7), the action (1) assumes the familiar Lorentz-invariant form in the new coordinates,

$$S = -\frac{1}{4g'^2}\int d^4x'\, F_{\mu\nu}^{a\prime}F_{\mu\nu}^{a\prime}, \qquad (8)$$

where

$$F_{\mu\nu}^{a\prime} = \partial'_\mu A_\nu^{a\prime} - \partial_\nu A_\mu^{a\prime} + f^{abc}A_\mu^{b\prime}A_\nu^{c\prime}. \qquad (9)$$

The coupling in the action (8) is not g but g' which is smaller by a factor of $\epsilon^{1/4}$. This means that the small parameter that controls the perturbative expansion in the theory (1) is not $\alpha_s = g^2/(4\pi)$ but rather

$$\alpha'_s = \frac{g^2}{4\pi\sqrt{\epsilon}}, \qquad (10)$$

which is much smaller than α_s, since ϵ is large.

Another way to derive Eq. (10) is by restoring $\hbar$ and c in the expression for the strong coupling constant α_s, which is given by $g^2/(4\pi\hbar c)$ in the vacuum. In our dielectric medium, the Coulomb potential between two static charges separated by r is $g^2/(\epsilon r)$. Thus, we have to replace g^2 by $g_{\rm eff}^2 = g^2/\epsilon$. The velocity of light c also needs to be replaced by the velocity of gluons v. This gives

$$\frac{g_{\rm eff}^2}{4\pi\hbar v} = \frac{g^2}{4\pi\sqrt{\epsilon}}, \qquad (11)$$

since $\hbar = 1$ in our unit system. Equation (11) coincides with α'_s in Eq. (10), as one expects.

Using Eq. (5), one can express the coupling α'_s in terms of the gap Δ,

$$\alpha'_s = \frac{3}{2\sqrt{2}}\frac{g\Delta}{\mu}. \qquad (12)$$

Equations (10) and (12) define the coupling in our effective theory at the matching scale with the original microscopic theory, i.e., at the scale Δ. The coupling increases logarithmically as one moves to lower energies, since pure SU(2) Yang-Mills theory is asymptotically free. This coupling becomes large at the confinement scale $\Lambda'_{\rm QCD}$, which is the mass scale of SU(2) glueballs. The spectrum of these glueballs is known from lattice studies of SU(2) Yang-Mills theory [8], except that the role of the speed of light is now played by v [Eq. (6)]. Since α'_s is tiny [because $\Delta/\mu \ll 1$ in Eq. (12)] it takes long to grow, and the scale $\Lambda'_{\rm QCD}$ is thus very small. Using the one-loop beta function, one can estimate

$$\Lambda'_{\rm QCD} \sim \Delta \exp\left(-\frac{2\pi}{\beta_0 \alpha'_s}\right) \sim \Delta \exp\left(-\frac{2\sqrt{2}\pi}{11}\frac{\mu}{g\Delta}\right), \quad (13)$$

where β_0 is the first coefficient in the beta function and is equal to 22/3 in SU(2) gluodynamics.

We can draw a few immediate conclusions from Eq. (13). First, $\Lambda'_{\rm QCD}$ depends very sensitively on the gap Δ, in particular on the numerical value of the constant b in Eq. (4). Unfortunately, the latter is not exactly known. The uncertainty in the value of the gap Δ translates into a huge variation of $\Lambda'_{\rm QCD}$. For example, if one uses

$$b = 512\pi^4\,, \quad (14)$$

which is obtained by solving the one-loop gap equation where the exchanged gluon propagator is replaced by the hard dense loop (HDL) expression [9], then with $\Lambda_{\rm QCD} = 200$ MeV we find $\Lambda'_{\rm QCD} \sim 10$ MeV at $\mu = 600$ MeV. However, if we use

$$b = 512\pi^4 \exp\left(-\frac{\pi^2+4}{8}\right), \quad (15)$$

which is obtained if one assumes the Bardeen-Cooper-Schrieffer ratio between the critical temperature T_c and the gap Δ, and computes T_c by taking into account the fermion wave-function renormalization [10], then, at the same chemical potential, $\Lambda'_{\rm QCD}$ is reduced to a mere 0.3 keV. Regretfully, neither Eq. (14) nor (15) seems to be entirely correct, since there are physical effects that they do not take into account (e.g., the Meissner effect). Clearly, any attempt to give even the roughest numerical estimate for $\Lambda'_{\rm QCD}$ requires an accurate determination of the gap Δ. It has been argued that to compute Δ one needs a better understanding of the issue of gauge invariance, the finite fermion lifetime, and the running of the coupling [11]. Regardless of all these uncertainties, the exponential dependence of $\Lambda'_{\rm QCD}$ on μ/Δ makes it safe to predict that, even at moderate values of μ, the confinement scale $\Lambda'_{\rm QCD}$ is very small, much smaller than $\Lambda_{\rm QCD}$.

Second, as the density, i.e., μ, is increased, $\Lambda'_{\rm QCD}$ vanishes exponentially fast due to the factor μ/Δ in the exponent. We arrive at the following picture of how deconfinement occurs at large densities. Strictly speaking, at any given density, the theory is confined. However, the confinement radius $1/\Lambda'_{\rm QCD}$ grows exponentially as the density is increased. Therefore, if one looks at the physics at some large, but fixed, distance scale, there is a crossover density when effectively the color degrees of freedom become deconfined at that scale. We call this phenomenon "asymptotic deconfinement."

The computation of ϵ and λ.—To find ϵ and λ, one has to calculate (1) by integrating out the quark degrees of freedom in the QCD Lagrangian. This amounts to computing the one-loop polarization operator $\Pi(q)$ and the gluon vertices $\Gamma_3(q_1,q_2)$, $\Gamma_4(q_1,q_2,q_3)$, etc. This procedure is the same as the one giving rise to the hard thermal loop (HTL) and hard dense loop effective actions [12]. The situation here is simpler than in the HTL and HDL cases: in the regime where all gluon momenta q are much smaller than Δ, the functions Π and Γ can be expanded in powers of q, yielding a local effective Lagrangian. (In contrast, the HTL and HDL actions are non-local, since the fermions do not have gaps.) The gauge invariance of the effective Lagrangian greatly simplifies our task: in order to know ϵ and λ, one needs to compute only the polarization tensor Π of the SU(2) gluons. The leading contribution at large density comes from the superconducting quark loop. The detailed calculation of Π was done in Ref. [5]. From Eq. (99a) of that paper one can derive the following expression for $\Pi^{00}_{ab}(q_0,\mathbf{q})$, $a,b=1,2,3$:

$$\Pi^{00}_{ab}(q_0,\mathbf{q}) = -\delta_{ab}\frac{g^2\mu^2}{\pi^2}\frac{\Delta}{q}\int_0^\infty dz \int_0^{q/2\Delta} dy \left(1 - \frac{z^2-y^2+1}{u_+u_-}\right) \times\left(\frac{1}{u_++u_-+q_0/\Delta} + \frac{1}{u_++u_--q_0/\Delta}\right), \quad (16)$$

where $u_\pm = \sqrt{(z\pm y)^2+1}$, and we assume that q_0 and $q \equiv |\mathbf{q}|$ are much smaller than μ so that the dominant contribution comes only from particles near the Fermi surface. Physically, the first term in parentheses is the probability to excite a quark-hole pair through an SU(2) gluon, and the last term contains the corresponding energy denominators for such an excitation. For $|q_0| > 2\Delta$, one should replace q_0 by $q_0 + i\epsilon$. Expanding Eq. (16) to quadratic order in q_0 and q around $q_0 = q = 0$, one finds

$$\Pi^{00}_{ab}(q_0,\mathbf{q}) = -\kappa\, q^2 \delta_{ab}\,, \quad (17)$$

where

$$\kappa = \frac{g^2\mu^2}{18\pi^2\Delta^2}\,. \quad (18)$$

The appearance of a factor μ^2 is due to the fact that the loop integral is dominated by the momentum region near the Fermi surface, whose area is proportional to μ^2. Similarly, one obtains from Eq. (99b) of [5] the following:

$$\Pi^{0i}_{ab}(q_0,\mathbf{q}) = -\kappa\, q^0 q^i \delta_{ab}\,. \quad (19)$$

The computation of $\Pi^{ij}_{ab}(q_0,\mathbf{q})$ is facilitated by writing

$$\Pi(q_0,\mathbf{q}) \equiv [\Pi(q_0,\mathbf{q})-\Pi(0,\mathbf{q})]+[\Pi(0,\mathbf{q})-\Pi_{\rm HDL}(0,\mathbf{q})] + \Pi_{\rm HDL}(0,\mathbf{q})\,, \tag{20}$$

where $\Pi_{\rm HDL}$ is the standard HDL gluon self-energy, which vanishes for $q_0=0$. The term $\Pi(0,\mathbf{q})-\Pi_{\rm HDL}(0,\mathbf{q})$ can be shown to be of order $\mathcal{O}(\Delta^2)$ and thus negligible compared to the first term $\Pi(q_0,\mathbf{q})-\Pi(0,\mathbf{q})$. The reason for the manipulation (20) is to get rid of the antiparticle contributions in this term. With Eq. (99c) of [5], the result for Π^{ij}_{ab} can be written analogously to Eq. (16) as

$$\Pi^{ij}_{ab}(q_0,\mathbf{q}) = -\delta_{ab}\frac{g^2\mu^2}{\pi^2}\frac{\Delta}{q}\int\limits_0^\infty dz\int\limits_0^{q/2\Delta}dy\int\limits_0^{2\pi}\frac{d\varphi}{2\pi}\,\hat{k}^i\hat{k}^j \times\left(1-\frac{z^2-y^2-1}{u_+u_-}\right) \times\left(\frac{1}{u_++u_-+q_0/\Delta}+\frac{1}{u_++u_--q_0/\Delta}-\frac{2}{u_++u_-}\right), \tag{21}$$

where $\hat{\mathbf{k}}=(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta)$, and $\cos\theta=2y\Delta/q$. Expanding to quadratic order in q_0 and q, one finds

$$\Pi^{ij}_{ab}(q_0,\mathbf{q}) = -\kappa\, q_0^2\delta^{ij}\delta_{ab}\,. \tag{22}$$

Note that (i) the polarization tensor satisfies current conservation, $q_\mu\Pi^{\mu\nu}_{ab}=0$; (ii) at this order, there is no spatial transverse contribution $\sim q^2\delta^{ij}-q^iq^j$ to Π^{ij}, although such a term is not forbidden by the symmetries.

After the quark loop has been integrated out, the quadratic term in the effective Lagrangian becomes $A^\mu(-q)[D^{-1}_{\mu\nu}(q)+\Pi_{\mu\nu}(q)]A^\nu(q)/(2g^2)$, where D is the bare gluon propagator. Comparing with the quadratic terms in Eq. (1), we obtain Eqs. (2) and (3). The fact that $\lambda=1$ [Eq. (3)] is due to the absence of the spatial transverse term in Π^{ij}.

In our analysis above we have neglected other low energy excitations: the unpaired fermions of the third color and the pseudoscalar isoscalar mode similar to the η meson. This is justified because they are colorless with respect to the unbroken $SU(2)_c$ gluons. It is also interesting to note that since the gap in the SU(2) colored quark spectrum, Δ, is much larger than $\Lambda'_{\rm QCD}$, the spectrum of mesons made of these quarks must resemble that of heavy quarkonia.

The asymptotic deconfinement phenomenon is specific to the case $N_f=2$. At sufficiently high densities when, effectively, $N_f=3$, the ground state is the color-flavor locked (CFL) state: the color symmetry is broken completely [13] and all gluons are screened, both electrically and magnetically [14]. The low energy modes in the CFL phase are the Goldstone modes described by a chiral effective Lagrangian [15,16]. Between the two regimes, when one has strange quark matter below the "unlocking" phase transition [17], the system might be an anisotropic dielectric if an ss condensate breaks rotational invariance. Another interesting regime with asymptotic deconfinement is that of high isospin density [18], where the gauge group of the gluodynamics of continuous media is SU(3).

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